Effects from the charm scale in $K^+ \to \pi^+ \nu \bar{\nu}$

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Abstract

We consider contributions to the rare decay $K^+ \to \pi^+ \nu \bar{\nu}$ which become non-local at the charm scale. Compared to the leading term, such amplitudes are suppressed by powers of m_K^2/m_c^2 and could potentially give corrections at the level of 15%. We compute the leading coefficients of the subleading dimension eight operators in the effective theory below the charm mass. The matrix elements of these operators cannot all be calculated from first principles and some must be modeled. We find that these contributions are likely to be small, but the estimate is sufficiently uncertain that the result may be as large as the existing theoretical uncertainty from other sources.

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The search for New Physics relies on experimentally accessible quantities whose Standard Model values can be predicted accurately and reliably. This task is often complicated by nonperturbative hadronic physics, especially when one is interested in the parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. To make progress, it is important to find processes where symmetry can be used to treat low energy QCD effects in a controlled and systematic way. One of these is the rare decay $K^+ \to \pi^+ \nu \bar{\nu}$. This process is an example of a neutral current $\Delta S = 1$ transition, which in the Standard Model can occur only via one-loop diagrams.

The leading contributions to the effective Hamiltonian for this decay are given by

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \Theta_W} \sum_l \left(V_{ts}^* V_{td} X(x_t) + V_{cs}^* V_{cd} X_{NL}^l(x_c) \right) \bar{s} \gamma^\nu (1 - \gamma^5) d \, \bar{\nu}_l \gamma_\nu (1 - \gamma^5) \nu_l \,, \quad (1)$$

where the index $l = e, \mu, \tau$ denotes the lepton flavor. The coefficient $X(x_t)$ arises from the top quark loop and is independent of lepton flavor. It is dominated by calculable high energy physics, and has been computed to $\mathcal{O}(\alpha_s)$ [1]. Because it grows as m_t^2, it is large and gives the leading contribution to the decay rate. If this were the sole contribution, the measurement of $K^+ \to \pi^+ \nu \bar{\nu}$ would yield a direct determination of the combination of CKM parameters $|V_{ts}^* V_{td}|$ [2]. However, due to the smallness of $V_{ts}^* V_{td}$ compared to $V_{cs}^* V_{cd}$, the charm contribution contained in the coefficient function $X_{NL}^l(x_c)$ is significant as well. These terms have been calculated to next-to-leading logarithmic order [3]. An important source of error in the calculation comes from the uncertainty in the charm quark mass, on which $X_{NL}^l(x_c)$ depends.

An important feature of the calculation is the fact that the hadronic matrix element $\langle \pi^+ | \bar{s} \gamma^\nu (1 - \gamma^5) d | K^+ \rangle$ is related via isospin to the matrix element $\langle \pi^0 | \bar{s} \gamma^\nu (1 - \gamma^5) d | K^+ \rangle$ responsible for $K^+ \to \pi^0 e^+ \nu$. This largely eliminates the uncertainty due to nonperturbative QCD, up to small isospin breaking effects [4]. However, there remain long distance contributions associated with penguin diagrams containing up quarks which can lead to on-shell intermediate states. Some of these have been estimated in chiral perturbation theory and found to be small [7]. The perturbative contribution from virtual up quarks is tiny, since it is suppressed compared to the charm contribution by m_u^2/m_c^2.

Summed over neutrino species, the branching fraction for $K^+ \to \pi^+ \nu \bar{\nu}$ is given by

$$B(K^+ \to \pi^+ \nu \bar{\nu}) = \kappa_+ \left[\left(\frac{\mathrm{Im}\, \xi_t}{\lambda^5} X(x_t) \right)^2 + \left(\frac{\mathrm{Re}\, \xi_c}{3\lambda^5} \sum_l X_{NL}^l(x_c) + \frac{\mathrm{Re}\, \xi_t}{\lambda^5} X(x_t) \right)^2 \right], \quad (2)$$

with $\lambda = \sin \theta_C \approx 0.22$ and

$$\kappa_+ = r_{K_+} \frac{3\alpha^2 B(K^+ \to \pi^0 e^+ \nu)}{2\pi^2 \sin^4 \Theta_W} \lambda^8. \quad (3)$$

Here $\xi_i = V_{is}^* V_{id}$, and r_{K+} absorbs isospin breaking corrections to the relationship between the decays $K^+ \to \pi^0 e^+ \nu$ and $K^+ \to \pi^+ \nu \bar{\nu}$ calculated in Ref. [4]. In terms of the Wolfenstein parameterization of the CKM matrix [5], the branching ratio may be written as

$$B(K^+ \to \pi^+ \nu \bar{\nu}) = 4.11 \times 10^{-11} \cdot A^4 X^2(x_t) \frac{1}{\sigma} \left[(\sigma \bar{\eta})^2 + (\rho_0 - \bar{\rho})^2 \right], \quad (4)$$
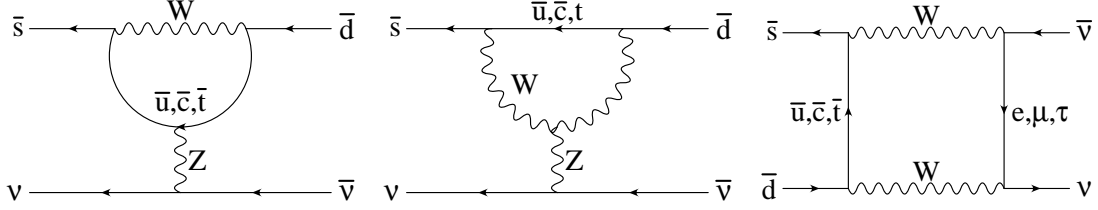
FIG. 1. Penguin and box diagrams responsible for $K^+ \to \pi^+ \nu\bar{\nu}$.

with

$$\sigma = (1 - \lambda^2/2)^{-2} \qquad \text{and} \qquad \rho_0 = 1 + \delta_c \,,$$

where δ_c absorbs the charm contribution. A measurement of the branching ratio then constrains the parameters $\bar{\rho}$ and $\bar{\eta}$, which are equal to the Wolfenstein parameters ρ and η up to known corrections of $\mathcal{O}(\lambda^2)$. The Alternate Gradient Synchrotron (AGS) experiment E949 at Brookhaven and the CKM collaboration at Fermilab propose to obtain measurements of the branching ratio for $K^+ \to \pi^+ \nu\bar{\nu}$ at the level of 30% and 10%, respectively. The Brookhaven experiment is the successor to AGS-E787, which saw one event in this channel [?]. These experimental prospects then fix the goal for the accuracy of the theoretical prediction at less than 10%.

The leading source of theoretical uncertainty is associated with the charm contribution. Calculations at next-to-leading order in QCD yield $\delta_c = 0.40 \pm 0.07$, where the error is due primarily to the uncertainty in the charm mass [3]. The errors from uncomputed terms of order $\alpha_s^2(m_c)$ are expected to be small. However, the computation of the charm contribution relies on an operator product expansion which is simultaneously a series in α_s and an expansion in higher dimension operators suppressed by powers of m_c. The operators which are of higher order in the $1/m_c$ expansion reflect the fact that the penguin loop becomes nonlocal at the relatively low scale m_c. One might expect the leading correction from higher order terms to give a contribution to δ_c of relative size $m_K^2/m_c^2 \sim 15\%$, large enough to affect in a noticeable way the extraction of $\bar{\rho}$ and $\bar{\eta}$ from the decay rate. It is important either to verify or to exclude the presence of new terms of such a magnitude.

In this note we will study the contributions of dimension eight operators to the decay $K^+ \to \pi^+ \nu\bar{\nu}$. We estimate the correction to δ_c and comment on the uncertainty induced. After discussing the relevant power counting, we present the calculation of the operator coefficients and an estimation of the correction to the decay rate. We will find a small contribution, but one that need not be negligible.

The decay $K^+ \to \pi^+ \nu\bar{\nu}$ proceeds via the loop processes shown in Fig. 1, which mediate the quark level transition $\bar{s} \to \bar{d}\nu\bar{\nu}$. These diagrams contain both short distance and long distance effects, which we separate by computing the effective Hamiltonian density $\mathcal{H}_{\text{eff}}$ at a low scale $\mu \lesssim 1\,\text{GeV}$. The effective Hamiltonian will receive corrections from the charm and top quarks, both of which have been integrated out of the theory, and from highly virtual up quarks. Soft up quarks remain in the theory, and are responsible for long distance corrections.

We construct the effective Hamiltonian with an operator product expansion. At leading order, the operator in $\mathcal{H}_{\text{eff}}$ which contributes to the decay is of dimension six,

$$O^{(6)} = \bar{s}\gamma^{\nu}(1 - \gamma^5)d\,\bar{\nu}\gamma_{\nu}(1 - \gamma^5)\nu. \tag{5}$$

The t quark contribution to the coefficient of this operator is obtained by evaluating the diagrams in Fig. 1 at the scale $\mu = M_W \approx m_t$ and matching on to the effective theory below this scale. At the same time, the W and Z are integrated out of the theory, producing four-fermion operators involving up and charm quarks as well. The charm contribution to the operator is then obtained by evaluating the diagrams contributing to the decay at $\mu = m_c$. These diagrams look like those in Fig. 1, but with the W and Z propagators replaced by local interactions.

Dimensional analysis indicates that the coefficient of the dimension six operator $O^{(6)}$ scales as $1/M_W^2$. The diagrams in Fig. 1 are quadratically divergent in the effective theory below M_W, and scale as Λ^2/M_W^4, where $\Lambda \sim M_W$ is an ultraviolet cutoff. The Glashow-Iliopoulos-Maiani (GIM) mechanism ensures that this leading divergence cancels, since it is independent of the mass m_q of the virtual quark. The consequence is that the coefficient of $O^{(6)}$ actually scales as m_q^2/M_W^4. In terms of the Wolfenstein parameter λ, the top coefficient has strength $\lambda^5 m_t^2/M_W^4$ and the charm coefficient has strength $\lambda m_c^2/M_W^4$. The top contribution is significant because of the large top mass, since $\lambda^4 m_t^2/m_c^2$ is of order 10.

For the purpose of power counting, the operators of dimension eight scale as

$$O^{(8)} \sim M_K^2 O^{(6)}, \tag{6}$$

appearing with generic coefficient $C_{(8)}$. Dimensionally, $C_{(8)}$ is proportional to $1/M_W^4$. The top contribution to $C_{(8)}O^{(8)}$ is suppressed by M_K^2/m_t^2 relative to its contribution to $O^{(6)}$, leading to an overall strength of order $\lambda^5 M_K^2/M_W^4$. The corresponding suppression for charm is only M_K^2/m_c^2, so the overall contribution of charm to $C_{(8)}O^{(8)}$ scales as $\lambda M_K^2/M_W^4$. Note that there is now no relative enhancement from the large top mass, so the top contribution to $C_{(8)}$ is suppressed relative to that of charm by λ^4 and can be neglected. Furthermore, the contributions in question are independent of m_q, so they cancel by the GIM mechanism when the up contribution is included.

However, the GIM cancellation is manifest in Feynman diagrams only for contributions which are perturbatively calculable. The long distance contributions involving soft up quarks will differ by factors of order one from their perturbative representations. For these parts of the diagrams, which scale as $1/M_W^4$, the GIM cancellation is ineffective. Such long distance contributions have been considered elsewhere [7], and estimated to be small. The GIM cancellation is also spoiled by logarithmic contributions proportional to $(M_K^2/M_W^4)\ln(m_c^2/M_K^2)$. Such terms may be generated by the running of $\mathcal{H}_{\text{eff}}$ between the scale m_c and the low energy scale $\mu \lesssim 1\,\text{GeV}$. This is not a large logarithm, numerically, but it allows us nonetheless to identify a GIM violating contribution to $\mathcal{H}_{\text{eff}}$. This term, which is generated by intermediate up quarks as shown in Fig. 2, is of the same power-counting size as the long distance contribution. But because the perturbative description of the long-distance part is inaccurate, there is no reason to expect the GIM cancellation to be restored when it is included.

The purpose of this paper is to compute the corrections to $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ of order $(M_K^2/M_W^4)\ln(m_c^2/M_K^2)$. These contributions are well defined, and it is important, in light of the experimental situation discussed above, to determine whether they introduce a theoretical uncertainty at a level competitive with the uncertainty due to m_c. Note that pure power
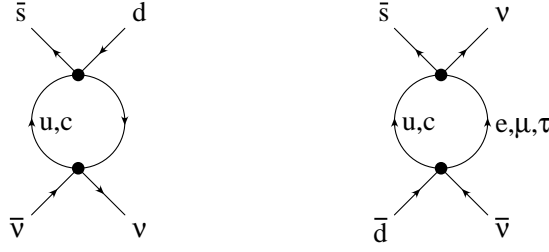
FIG. 2. Diagrams leading to operators of dimension eight.

counting arguments permit a relative contribution to δ_c of the order of $(m_K^2/m_c^2)\ln(m_c^2/\mu^2)$, which could be as large as 20%, depending on the value chosen for the hadronic scale μ.

We will study the effective Hamiltonian of dimension eight operators, at leading order in α_s. This Hamiltonian receives logarithmically enhanced contributions from the up quark loops in Fig. 2. We also must consider the matching corrections at the scale $m_c \approx m_\tau$, when the tau lepton is integrated out of the theory. Because the matching function $F(m_c/m_\tau)$ cannot be approximated by an expansion in m_c/m_τ, the combination $[F(m_c/m_\tau) - F(m_u/m_\tau)]$ is a GIM violating finite matching correction which also must be included.

The effective Hamiltonian density at the scale μ takes the form

$$\mathcal{H}_{\text{eff}} = \sum_{l,i} C_i^l(\mu) O_i^l(\mu) \,, \tag{7}$$

where l denotes lepton flavor. As it turns out, there will be two dimension eight operators generated in the theory below m_c,

$$O_1^l = \bar{s}\gamma^\nu(1-\gamma^5)d\,(i\partial)^2\left[\bar{\nu}_l\gamma_\nu(1-\gamma^5)\nu_l\right],$$
$$O_2^l = \bar{s}\gamma^\nu(1-\gamma^5)(iD)^2 d\,\bar{\nu}_l\gamma_\nu(1-\gamma^5)\nu_l + 2\bar{s}\gamma^\nu(1-\gamma^5)(iD^\mu)d\,\bar{\nu}_l\gamma_\nu(1-\gamma^5)(i\partial_\mu)\nu_l$$
$$+ \bar{s}\gamma^\nu(1-\gamma^5)d\,\bar{\nu}_l\gamma_\nu(1-\gamma^5)(i\partial)^2\nu_l \,. \tag{8}$$

The first of these operators does not receive any logarithmic QCD corrections below the scale m_c, because it is proportional to a current which is partially conserved. The second does, but we will not include higher order corrections of relative order $\alpha_s \ln(m_c/\mu)$. Note that this is not inconsistent with resumming terms of order $\alpha_s^n \ln^n(M_W/m_c)$.

The operators $O_{1,2}^l$ are generated by the diagrams in Fig. 2. In principle, one might have expected the diagrams in Fig. 3 to generate additional operators with a gluon field strength, such as

$$O_3^l = \bar{s}\gamma^\nu\sigma^{\alpha\rho}G_{\alpha\rho}(1-\gamma^5)d\,\bar{\nu}_l\gamma_\nu(1-\gamma^5)\nu_l \,. \tag{9}$$

However, it turns out that contributions to all such operators cancel.

The operators of dimension six that will induce $O_{1,2}^l$ in $\mathcal{H}_{\text{eff}}$ are

$$O_4 = \bar{s}\gamma^\nu(1-\gamma^5)d\,\bar{u}\gamma_\nu(1-\gamma^5)u\,,$$
$$O_5 = \bar{s}\gamma^\nu(1-\gamma^5)u\,\bar{u}\gamma_\nu(1-\gamma^5)d\,,$$
$$O_6^l = \bar{u}\gamma_\nu\left(-\tfrac{4}{3}\sin^2\theta_W(1+\gamma^5) + (1-\tfrac{4}{3}\sin^2\theta_W)(1-\gamma^5)\right)u\,\bar{\nu}_l\gamma^\nu(1-\gamma^5)\nu_l\,,$$
$$O_7^l = \bar{s}\gamma^\nu(1-\gamma^5)u\,\bar{\nu}_l\gamma_\nu(1-\gamma^5)l\,,$$
$$O_8^l = \bar{u}\gamma^\nu(1-\gamma^5)d\,\bar{l}\gamma_\nu(1-\gamma^5)\nu_l\,. \tag{10}$$
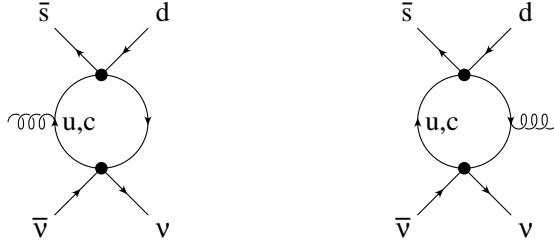
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FIG. 3. Diagrams which could lead to an operator with a gluon field strength.

The operator O_6^l comes from virtual Z exchange, the others from W exchange. The renormalization group equations for O_1^l and O_2^l are

$$\mu \frac{dC_1^l}{d\mu} = = \gamma_1 C_4 C_6^l + \gamma_1' C_5 C_6^l \,,$$

$$\mu \frac{dC_2^l}{d\mu} = = \gamma_2 C_7^l C_8^l + \sum_j \gamma_{2j} C_j^l \,. \tag{11}$$

The anomalous dimensions γ_1, γ_1' and γ_2 are of order one. The matrix γ_{2j} is of order α_s and comes from QCD running below m_c; it will not be included in our analysis.

Computing the diagrams in Fig. 2 and solving the renormalization group equations, we find the coefficients at the scale μ,

$$C_1^{e,\mu,\tau} = \frac{c_0}{6M_W^2}(1 - \tfrac{4}{3}\sin^2\Theta_W)G(\alpha_s)\log(\mu/m_c)\,,$$

$$C_2^{e,\mu} = -\frac{c_0}{M_W^2}\log(\mu/m_c)\,, \tag{12}$$

$$C_2^\tau = -\frac{c_0}{4M_W^2}f(m_c^2/m_\tau^2)\,,$$

where

$$c_0 = \frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\theta_W}V_{cs}^*V_{cd}\,,$$

$$f(x) = \left(\frac{6x-2}{(x-1)^3} - 2\right)\log x - \frac{4x}{(x-1)^2}\,, \tag{13}$$

$$G(\alpha_s) = 2\left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right)^{-6/25}\left(\frac{\alpha_s(m_b)}{\alpha_s(M_W)}\right)^{-6/23} - \left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right)^{12/25}\left(\frac{\alpha_s(m_b)}{\alpha_s(M_W)}\right)^{12/23}\,,$$

and we have used $V_{us}^*V_{ud} \approx -V_{cs}^*V_{cd}$. Taking the values $m_c = 1.3\,\text{GeV}$, $m_b = 4.5\,\text{GeV}$, $\Lambda_{\overline{MS}} = 0.35\,\text{GeV}$ and $\mu = m_c/2$, we find

$$C_1^{e,\mu,\tau} = 0.05 \cdot c_0/M_W^2\,, \qquad C_2^{e,\mu} = 0.69 \cdot c_0/M_W^2\,, \qquad C_2^\tau = 0.28 \cdot c_0/M_W^2\,. \tag{14}$$

By comparison, the coefficient of the leading charm contribution in Eq. (1) is given by $X_{NL}^l(x_c)\,c_0$, which is $4.0\,m_c^2 \cdot c_0/M_W^2$ for $l = e,\mu$ and $2.7\,m_c^2 \cdot c_0/M_W^2$ for $l = \tau$.

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To compute the contribution to the decay rate, we also need the matrix elements $\langle \pi^+ \nu_l \bar{\nu}_l | O_{1,2}^l | K^+ \rangle$. The leading relative corrections come from the interference of $O_{1,2}$ with $O^{(6)}$ and depend on the ratios

$$R_i = \frac{\text{Re} \int d[\text{P.S.}] \, |\langle \pi^+ \nu_l \bar{\nu}_l | O_i^l | K^+ \rangle^* \langle \pi^+ \nu \bar{\nu} | O^{(6)} | K^+ \rangle}{\int d[\text{P.S.}] \, |\langle \pi^+ \nu \bar{\nu} | O^{(6)} | K^+ \rangle|^2} \, . \tag{15}$$

The matrix element of the operator O_1 is easy to calculate, since it depends only on the lepton momenta. The leptons are treated perturbatively, so the hadronic dependence of the matrix element of O_1 is the same as that of $O^{(6)}$. We then find

$$R_1 = \langle (p_\nu + p_{\bar{\nu}})^2 \rangle = (180 \, \text{MeV})^2 \, . \tag{16}$$

Unfortunately, the matrix element of O_2 cannot be calculated analytically, since it involves the gluon field through the covariant derivative acting on the down quark. We are forced to rely instead on model dependent estimates, which are notoriously unreliable. One ansatz would be to take

$$\langle \pi^+ \nu \bar{\nu} | O_2 | K^+ \rangle_\mu \approx \mu^2 \, \langle \pi^+ \nu \bar{\nu} | O^{(6)} | K^+ \rangle_\mu, \tag{17}$$

or $R_2 \approx \mu^2 \sim (650 \, \text{MeV})^2$. Another would be to neglect the gluon field and model the matrix element as

$$\langle \pi^+ \nu \bar{\nu} | O_2 | K^+ \rangle = (p_\pi + p_{\bar{\nu}})^2 \, \langle \pi^+ \nu \bar{\nu} | Q^{(6)} | K^+ \rangle, \tag{18}$$

in which case

$$R_2 \approx \langle (p_\pi + p_{\bar{\nu}})^2 \rangle = (340 \, \text{MeV})^2 \, . \tag{19}$$

Of course, neither of these guesses need be correct within better than an order of magnitude. Fortunately, lattice QCD methods are advancing quickly, to the point that a true unquenched lattice calculation of this matrix element may soon be feasible. For now, we will take these two crude guesses to bracket roughly the actual value of R_2.

We now write the branching fraction for $K^+ \to \pi^+ \nu \bar{\nu}$ as in Eq. (4), with

$$\rho_0 = 1 + \delta_c (1 + \delta_8) \, , \tag{20}$$

where δ_8 is the new term which we are computing. Summed over lepton species, the contribution of charm at dimension six is given by

$$\delta_c = \frac{P_0(x_c)}{A^2 X(x_t)} = \frac{1}{3\lambda^4} \sum_l X_{NL}^l(x_c) \cdot \frac{1}{A^2 X(x_t)} \, . \tag{21}$$

A next to leading order analysis yield $P_0 = 0.42 \pm 0.06$, where the error arises in large part from the uncertainty in the charm quark mass [3]. This value of P_0 gives $\delta_c = 0.40 \pm 0.07$, where we use $X(x_t) = 1.53 \pm 0.01$ and $A = 0.83 \pm 0.06$. The fractional correction due to dimension eight operators is then

$$\delta_8 = \frac{1}{3 P_0 \lambda^4} \sum_l \left(C_1^l R_1 + C_2^l R_2 \right) = \delta_8^{(1)} + \delta_8^{(2)} . \tag{22}$$

The first term, for which the matrix element is calculable, is negligible in size: with our choice of inputs, $\delta_8^{(1)} = 5.6 \times 10^{-5}$. The second, highly uncertain, term is much bigger, with $\delta_8^{(2)}$ between 1% and 5% for our adopted range for R_2. On the one hand, even δ_8 as large as 5% is somewhat below the existing uncertainty on δ_c from the value of m_c. On the other, if our "upper limit" on the matrix element of O_2^l were too small by even a factor of two, which need not be unlikely, these contributions would have a significant effect on the extraction of CKM parameters from the branching fraction.

We have made a number of approximations in obtaining these results. A potentially important one, within the perturbative calculation, is that we have neglected QCD running below m_c. We could include these QCD corrections for O_1 simply by incorporating the known running of the coefficients C_4 and C_5; doing so decreases C_1 by a factor of two. However, the running of O_2 is not equally trivial, since O_2 itself is renormalized in QCD. In view of the large uncertainty in the matrix element of O_2, including these QCD corrections would not at this time increase the reliability of our prediction.

Of course, the key uncertainty arises not from QCD perturbation theory but from the actual value of $\langle \pi^+ \nu \bar\nu | O_2 | K^+ \rangle$. Only a realistic lattice computation will settle the matter. We would argue, in fact, that such a calculation is really required for one to be confident that the effects we have considered do not spoil the extraction of CKM matrix elements from the proposed experiments on $K^+ \to \pi^+ \nu \bar\nu$. This is not the only case where higher dimensional operators can play an interesting role in kaon decays [8,9].

In summary, we have computed the dominant contribution to the coefficients of dimension eight operators contributing to the decay $K^+ \to \pi^+ \nu \bar\nu$. Our best estimate is that this represents a correction of no more than 5% to the leading charm contribution to the decay. However, our ignorance of relevant hadronic matrix elements leaves open the possibility that these contributions could represent an uncertainty as large as or larger than that due to the charm quark mass. A lattice calculation of nonperturbative corrections, and to a lesser extent the inclusion of perturbative QCD corrections below the charm scale, will be indispensable to reducing this uncertainty before the planned experiments begin to take their data.

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